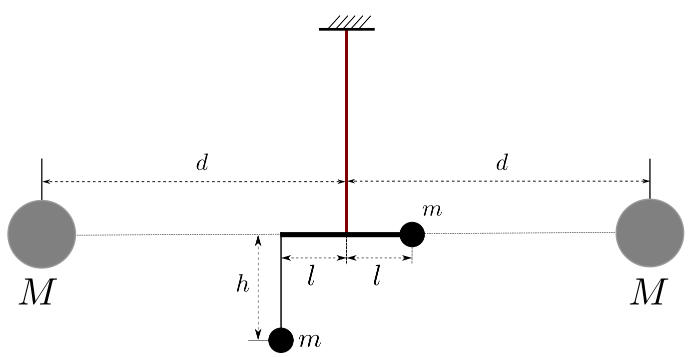
THE 50th—22nd INTERNATIONAL—RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS

25 October— 4 November 2019

In association with UNESCO, the world's scientific community commemorates this year the 100th anniversary of the death of Roland Eötvös (1848-1919), a pioneer of high precision gravitational physics, founding father of geophysics and innovator of higher education. For further references see: Roland Eötvös Commemorative Year.

In this centenary year, as part of a series of scientific events and exhibitions, the Ortvay Competition was organized to commemorate Eötvös. One of the aims of the organizers was to present some problems related to Eötvös's works which are still up to date, internationally significant both in theory and in application. What's more, it's a rare coincidence that this year is an exceptional anniversary event since the Ortvay Competition is now held for the 50th time.

1. Between two bodies of mass M, which are at distance $2d$ from each other, an Eötvös pendulum is placed in the middle, such that the pendulum makes low amplitude torsional vibrations. In the Eötvös pendulum, two masses m are attached to the two ends of the horizontal bar of length $2l$, such that one of these hangs on a string of length h. A thin wire is attached to the middle of the horizontal bar, such that it is perpendicular to the line between the two masses M. The torsion constant of the wire is D. Determine the period of the torsional vibrations, that is, the period of oscillating rotation around the axis defined by the wire! Does the period time depend on h? Assume that $l, h \ll d$ and $m \ll M$!



(József Cserti)

2. At the University of Science in Budapest which currently carries the name of Eötvös, the instrument shown in the figure was designed and built by Loránd Eötvös, in the physics building (D) completed by 1886, with the purpose of measuring the gravitational constant. He has modified the original Cavendish experiment so that the small masses m at the ends of the light aluminum bar, suspended with a torsional wire, are not displaced by the attractive force of masses at the same height. In his case, the two large balls of mass M, rotatable in a horizontal plane, are placed below the supporting aluminum rod, according to the figure. Let us denote the distance between the centers of the small masses by d, and let us set the distance of the centers of the larger masses also to d. Let us denote the vertical distance between the mid-points of the small and large mass pairs by h. Determine the angle between the horizontal plane and the line connecting one smaller mass and the closer large mass, in the case when the torque excerted by gravitational attaction is the largest!

With numerical calculations, determine how this angle depends on h. Estimate this angle for the limit of $R < h \ll d$ where R is the radius of the larger balls!

The source of the figure is the publication of Loránd Eötvös in the Annalen der Physik und Chemie in 1896, which was reproduced by Pál Selényi in 1953, in his Collected Works of Eötvös.

(Gyula Radnai and József Cserti)

3. The first internationally accepted scientific results of Loránd Eötvös treated the concept of surface tension. He elaborated a new method to measure the surface tension and gave the relation between molecular weight and surface tension of different materials (Eötvös law).

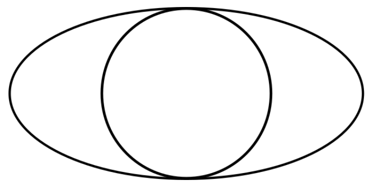
The surface tension tries to smooth the surface of fluids thus the deformation of the surface propagates away as a wave. Theese short wavelength capillary waves are not to be mistaken for the gravity waves of fluid surfaces, governed by the gravitational force. It is easy to measure on the water surface that the velocity of the capillary waves with wavelength about 1 cm is approximately 10 cm/s.

One of the interesting properties of this capillary waves is similar to Kepler's third law of planetary motion: the cube of wavelength λ of capillary waves is proportional to the square of the period T of the waves.

Based on these information estimate the size of the molecules!

(Miklós Vincze and Gyula Dávid)

4. The observed sea level acceleration (the vectorial sum of the gravity and the centrifugal force) varies from point to point at the surface of the Earth; it is maximal on the poles, while the minimum value is observed at the Equator. This variation was first observed by the French astronomer Jean Richer as early as 1672. He calibrated his pendulum clock at Paris and used again in Cayenne, in South America. For this exercise, let us approximate the latitude of Paris as 45 degrees (instead of the real 49) and the one of Cayenne as 0 degrees (instead of the real 5). He observed that the pendulum clock goes systematically inaccurate. This observation was acknowledged by R. Eötvös as one of the most important experiments concerning the shape of the Earth.



The difference is the sum of three effects: 1) The shape of the Earth is (almost) a spheroid (ellipsoid of revolution), so the pole is closer to the center of mass than the Equator; 2) the centrifugal force (due to the rotation of Earth around its axis) is zero at the pole and shows maximum at the Equator; and 3) there is a mass excess (in the above figure between the 'sphere' and the 'ellipsoid') and its gravity occurs stronger around the Equator. The questions (let's suppose the pendulum clock behaves as ideal mathematical pendulum at very small angles):

a) Was the clock of Richer, calibrated in Paris, late or early in Cayenne?

b) Estimate the daily inaccuracy in seconds.

c) Make a quantitative comparison (separately) of the above three effects on the observed gravity accelerations between the pole and the Equator.

(Gábor Timár and László Szarka)

5. A ballistic missile, launched from a point of Earth's surface, with an initial velocity v_0 below the escape velocity at an angle α will eventually reach Earth after moving along an elliptical trajectory.

a) Give the parameters of the elliptical trajectory!

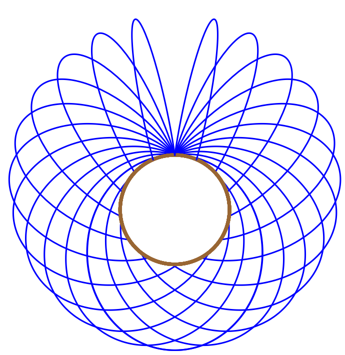
b) Determine the flight time and distance! How far is the highest point from the Earth's surface?

c) How much the initial velocity should be for a missile launched at given angle α, so that the impact point is exactly on the opposite point of the Earth's surface relative to the launch site? What is the time of flight? Determine the possible angles α.

d) How should one choose the launch angle starting with v_0 so that the missile impacts in the shortest time after departure? (The angle α of launch is positive.)

e) The figure below shows the trajectories of missiles launched at various angles with a given initial velocity v_0. Determine the envelope curve of the trajectories.

Assume that the rocket moves ballistically, and neglect effects of air drag.



(József Cserti and Gyula Dávid)

6. Calculate the length of the skid marks on the runway made by the rear wheels of a landing aircraft, and the total energy spent on wearing the tires while skidding! Use the following assumptions. Before landing, the wheels are not rotating. The touchdown speed is 252 km/h, the landing angle is 3 degrees with respect to the horizontal runway, the mass of the aircraft is 100 tons. After full stopping of the aircraft, the length of the springs pushing the wheels from above changes by 0.5 m compared to the length before landing. The wheel diameter is 70 cm, and there are 12 rear wheels next to each other, 100 kg each. The coefficient of dynamic friction is 0.7. While the wheels are skidding, the captain does not break the wheels, and does not make maneuvers with the wings either.

(Gábor Veres)

7. There are two masses, m_1 and m_2 on a table. An ideal thin filament bounds them. Initialy the filament is straight, and it touches a vertical nail fixed to the table. At start, the mass, m_1 has a velocity orthogonal to the filament, so the line of the filament is broken at the nail. Give the orbit of the masses. There is no friction anywhere.

(László Holics and Géza Tichy)

8. Consider a 'rimless wheel', which consists of $N > 4$ thin radial homogeneous rods, similar to the spokes of a normal wheel. The rods are fixed to each other such that their ends point to the vertexes of a regular polygon. This object is placed on a slope with angle α. Which is the minimal coefficient of friction, such that the body does not slip during rolling down?

(Ákos Gombkötő)

9. A cross-country skier can climb upwards with his feet slightly sideways holding the skis in a V shape. With a fast motion, the skier stands alternatively on one of her/his foots and the other. When the skier leans out best in one direction, she/he steps forward s with the other leg, then she/he does the same motion leaning in the other direction, repeatedly.

What is the possible angle of the slope the skier can climb up if her/his legs can be held sideways at best by angle γ and separated by distance d? The skier's center of gravity is at a height h from the ground.


It is also limited how fast the skier can stand on her/his leg alternatively, let the maximum frequency of this motion be f. Assume that the ski can slide practically without friction in the longitudinal direction, but laterally the coefficient of friction is very large! Is it possible to overcome any elevation angle if the frequency is large enough?

How does the amplitude of the right-to-left leaning of the skier depend on the elevation angle? Does the speed of the skier depend on the elevation angle?

(Zoltán Kaufmann)

10. Let us assume, that we overdo the climate protection actions, the world's oceans freeze, and the Great Oceanic Conveyor Belt stops. How much, and in which direction will the Earth's rotation period change?

(Gyula Dávid)

11. Describe the motion of the free, most symmetric top in four dimensions (4D). Devise a useful visualization, this is part of the task.

    Hint: Write up and study the 4D Euler equation for the most symmetric non-spherical rigid body.

(Géza Györgyi)

12. A small amplitude, but otherwise arbitrary, transversal wave-packet travels along a long rope. The rope is under a strong tension, and has a finite weight. The force which acts on the end of the rope is proportional to the velocity. Which is the proportionality factor, if no waves are reflected from the end of the rope?

(Péter Gnädig)

13. A flexible, elastic rod of uniform cross section, with one end clamped and the other one free, performs harmonic, damped oscillations in the plane, without getting twisted. Let dissipation be due to the internal viscous stress proportional to the rate of deformation. Give a method to determine the frequencies and quality factors of the modes, possibly explicit results for the first two modes, in the limit of weak damping for all modes, as well as some numerical results in a practical set-up and with realistic material constants. Study only the transversal oscillations. Gravity is negligible.

(Géza Györgyi)

14. On calm, warm, cloudless summer days one can observe, that the sound of the passenger-planes flying high becomes audible practically all of a sudden, instead of getting strong gradually. What is the cause of the phenomenon? Estimate, where above the horizon should one look for a plane flying right toward him, if its sound just became audible.

(Ferenc Woynarovich)

15. In a parallel universe, the water of the river Danube flooded the basement of the adjacent University building. The rescue personnel promptly descended to study the situation. Before actually entering the water, a small underwater ring appeared sidewards in the light of their electric torch, which was probably dropped by one of the female visitors of the Resarcher's Night. As the two firefighters has also participated the popular lecture series 'From Atoms to Stars' at the University, and got attracted to physics, they started to make actual measurements. One of them chose two rays emerging from a single point of the ring, such that the rays are in the same vertical plane close to each other. He determined the depth of the ring under the water, based on the distance and the angle between the rays. The other did a similar excersise, however he chose two closeby rays, which were in different vertical planes, but had the same angle relative to the horizontal plane. This depth value was different from the first one.

    Explain the reason for this difference, by calculating the depth values in both cases. Assume that the ring was spotted (seen from above the water) at an angle α relative to the vertical direction, and that it was actually in depth h. Assume that the ring is pointlike. Consider however the fact that the water could have dissolved a considerable amount of material, such as the salt stored in the basement for de-icing purposes, which arranged according to density. This means that the refraction index of the water depends only on the vertical coordinate, according to a fixed and known function.

(József Cserti and Zoltán Kaufmann)

16. As it is known (Ortvay contest, 1991, Problem 25.), the people of Atlantis were living under water. For the Atlanteans, it was important to estimate the storminess of the water surface above them, that is, what is the wave steepness. For this, they worked out an exclusively optical method: directing their instrument upwards, they measured the intensity distribution of the light originating from the free sky as a function of zenith angle. Snapshots would show that there is a specific critical zenith angle value θ_c, above which practically no light arrives, whereas from lower angle, a complicated pattern appears due to the waves.

    Performing the time averageing of the intensity, the complicated pattern is simplified, and if all light which is reflected from the surface of the waves is filtered out, then the angle θ_c is easy to register. From this value, the typical maximal slope of the waves can be calculated.

    Let us assume that the illumination from the full sky is measurable.

    The questions are the following:

    a) Determine the correspondence between the steepness of the waves, and the viewing angle! Assume for simplicity that the waves are sinusoidal.

    b) Considering that Atlantis (as known) was in a strait between the Falkland Islands, a considerable fraction of the waves arrived from a given direction. Study in this one-dimensional situation the time dependence of the angular region shape with no illumination!

    (Ákos Gombkötő)

17. Various problems at the Ortvay Contest were dealing with optical caustic. The simplest case is when a broad, parallel beam arrives to our cup from sideways, resulting in illuminated curves on the bottom of the cup, or on the surface of our coffee. Such caustics consist of high intensity (in the limiting case of geometrical optics, infinitely intensive) points and lines. Let us study for simplicity the projection of the light rays on a horizontal surface. The caustic results as the envelope of the reflected rays, which comes from the circular side wall of the coffee cup in the example above.

    Let us study now the inverse problem, that is, what should be the reflective surface / coffee cup side wall shape, to produce a caustic of a given shape!

    a) First find the reflective surface producing a semi-circle caustic.

    b) Try to solve the problem generally!

    c) Which is the reflective surface creating a heart shaped caustic?

    (Zoltán Kaufmann)

18. The center of the galaxy is teeming with stars. There is a black hole at the center of mass $4 \times 10^6\, M_\odot$, which is surrounded by a spherical population of 1 million stars in a radius of 1 parsec. The stars' number density is propotional to $r^{-1.75}$ in this region where r is the distance from the center, and their mass distribution is proportional to $m^{-2.3}$ where $0.2\, M_\odot < m < 50\, M_\odot$. The stars' luminosity as a function of their mass is $(m/M_\odot)^{3.5} L_\odot$ where $L_\odot = 3.8 \times 10^{26}$ W and $M_\odot = 2 \times 10^{30}$ kg.

    a) Imagine the sky as seen on the surface of a planet that is identical to the Earth but which orbits the central black hole at distance r. How many stars are visible to the naked eye?

    b) Determine the mean temperature on the planet's surface as a function of r between 0.001 and 1 parsec.

    c) What is the probability for water to be in the liquid phase on the surface of the planet as a function of r?
    Neglect the interstellar gas and gravitational lensing. For the probability calculation draw the positions and the masses of the stars randomly from the distributions given above.

    (Bence Kocsis)

19. The full Moon is sometimes visible during daylight in the sky. Estimate how much brighter is the full Moon's surface as observed from the Earth's surface relative to the daytime sky.

Useful data: the Moon's geometric albedo is 0.12, its spectrum is identical to that of the Sun. The Sun emits a black body radiation of temperature 5800 K. Calculate the brightness of the daytime sky as a function of direction relative to the Sun, in the limit that the gas particles in the atmosphere scattering the sunlight have a much smaller size than the wavelength λ of the light. Neglect multiple scattering and the light reflected from the Earth's surface. Assume that the atmosphere is spherically symmetric and that its temperature and chemical composition are independent of elevation, 250 K and 29 g/mol, respectively. Give the result for $\lambda = 400$ nm and 700 nm.

(Bence Kocsis, András Pál and Gábor Horváth)

20. One of the planets of the Little Prince (orbiting on a circular trajectory around its star) has radius R and made of homogeneous dark solid rock (which can be considered as a blackbody) of density ϱ, specific heat capacity c and heat conductivity κ. A beam of approximately parallel light rays of uniform intensity I falls onto the planet from its central star, which illuminates half the surface of the planet.

We would like to investigate the daily variation of temperature at a given point on the surface of the planet. In order to do this, let us use a coordinate system with axes fixed relative to the star, in which the planet rotates. Choose the axis x so that it is always parallel with the direction of the incident light rays and it passes through the center of the planet. Due to the circular motion of the planet around the star and the rotating motion of the planet around its axis, the planet rotates with angular speed Ω around axis z which is perpendicular to axis x. (In other words, Ω is the angular speed of the apparent motion of the star seen from the planet.)

Calculate and visualize on an appropriate diagram the surface temperature $T(\theta, \varphi)$ of the planet as a function of the usual spherical coordinates θ measured from the axis z and φ measured from the axis x. Assume that the surface temperature $T(\theta, \varphi)$ has reached its stationary distribution.

(Máté Vigh)

21. The horizontal surface of a solid body is exposed to a laser light from above, i.e., the incident ray is perpendicular to the surface. The intensity of the laser beam follows Gaussian profile, i.e., it is proportional to $\sim e^{-r^2/(2r_0^2)}$ where the variable r is the distance measured from the middle line of the laser beam and $r_0 = 7\cdot 10^{-3}$ m is the lengthscale characterizing the size of the laser spot. The intensity exhibits an exponential decay inside the solid body perpendicularly to the horizontal surface. The exponential decay is determined by the penetration depth $l_0 = 3\cdot 10^{-3}$ m. The laser heats the solid body in such a way that the volumetric heat production rate is proportional to the local intensity. The total power heating the solid body is 0.7 W.

Inside the solid material, the heat is transferred through conventional heat conduction governed by Fourier's law. The heat conductivity of the material is $2.1\,\mathrm{W/(m\cdot K)}$ which does not vary with the temperature. The solid body emits heat to the environment through the horizontal surface. The heat transfer coefficient of this linear process is constant on the whole surface (including the area of the laser spot) and has a value of $40\,\mathrm{W/(m^2K)}$. The horizontal surface can be considered infinitely large and the downward, vertical extension of the solid body can also be assumed infinitely large as well.

After switching on the laser, we wait until the steady state is reached. The environmental temperature is 300 K and does not change with time.

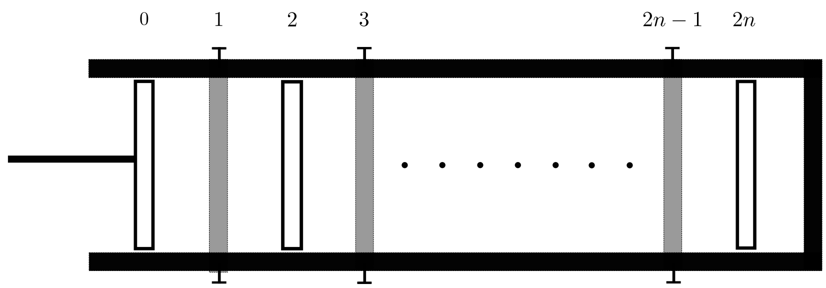
a) What is the temperature on the horizontal surface in the centre of the laser spot when the steady state is reached?

b) What is the maximal value of the temperature inside the solid body when the steady state is reached and where does the temperature take its maximum?

c) Is it possible to choose the parameters of the experiment such that the maximal temperature is measured on the horizontal surface?

(Ádám Bácsi and Örs Sepsi)

22. A thermally insulated container is divided into $2n+1$ equal sections by $2n$ walls, each containing monatomic ideal gas with the same pressure and temperature. The walls with odd numbers (from 1 to $2n-1$) are fixed and good heat conductors, the walls with even numbers (from 2 to $2n$) are thermally insulating and freely moving like pistons. Similarly the 0th wall is thermally insulating and is moveable. This gas in the first section of the container is slowly being compressed by moving the 0th wall. How does the temperature of the gas in this section change as a function of its volume? What can we say about the displacements of the freely moving walls?



(Gergely Fejős)

23. The *ordinary dipole-skinned* (*Dipoltergus Absolutiensis L.*) is the Heraldic beast of Gummy Coast. It has the peculiar property that on his skin, there are thin, low density, longitudinally elastic hair, with a specially modified cell at the end of each filament. If the animal gets scared or is angry, on his skin an electric dipole distribution appears with approximately homogeneous dipole density, oriented ortogonally to the surface of the skin. At the same time, a charge is transferred to the end of the hair filaments, such that due to the interaction between the charge and the dipoles, the hair is repelled away from the skin. In this 'blown-up' state of the animal, the manifold of hair filaments is in thermal equilibrium. We can neglect the interaction between the fibers, as well as gravity effects, and we can assume the filaments to be one dimensional.

Determine the average distance between the end of the filaments from the skin. How much is the energy increase which can be assigned to the fur of the animal due to this physiological change?

(Ákos Gombkötő)

24. A sphere of radius R is filled with low pressure ideal gas, such that the mean free path is $l \approx R$. Inside the sphere, at a distance h from its center, a small pressure sensor is placed, with its radius $r \ll R$. From the sensor, the gas molecules are bouncing off perfectly elastically. The detector calculates the pressure p by averageing over a very long time period, longer than any relevant characteristic time parameters. Determine the measured pressure value p as a function of h.

(Ákos Gombkötő)

25. In the field of physical chemistry, the study of the rotational states of a given molecule is important and regularly addressed. To describe it, the classical physical pendulum is used as a baseline model.

Starting from this, let us find the equations which can be used to describe the rotational states of a given molecule, assuming that it interacts thermally with the other molecules! Besides, study numerically the solutions in the limits $T \to 0$, and $T \to \infty$!

Hint: In order construct a model for the rotating system, take a planar pendulum, and consider the environment of the system as an ensemble of infinitely many harmonic oscillators!

(Balázs Endre Szigeti)

26. Determine the equation of a single field line (different from the straight line) of a pointlike dipole in the simplest possible way!

(Gyula Radnai)

27. Consider a thin, planar conductive body with the shape of a Cornu-spiral. The object is electrically charged. Give the resulting charge density distribution.

    Hint: It is useful to introduce an appropriate coordinate system.

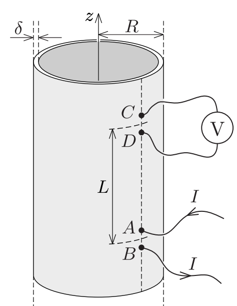
(Ákos Gombkötő)

28. A rotating, solid, conductive sphere is filled with gas. The polarizability of the gas is proportional to the pressure. Estimate the electric capacitance of the sphere as a function of the angular velocity of the rotation. The effect of gravity can be neglected.

(Ákos Gombkötő)

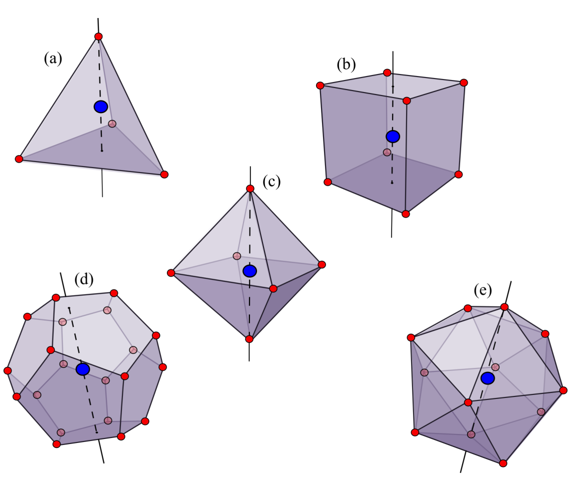
29. A very long cylindrical shell of radius R and thickness δ ($\delta \ll R$) is made of uniform conducting material of resistivity ϱ. The axis of the cylinder is the vertical z axis. We let current I in at point A and out from point B, where both A and B located on the same generatrix of the cylinder, and the distance AB is $d \ll R$ (while $\delta \ll d$).

    Find the voltage between points C and D (separated by distance d) located above points A and B at distance L ($L \gg d$).



(Máté Vigh)

30. Let us place to the vertices of each Platonic solid (that is, (a) tetrahedron, (b) cube, (c) octahedron, (d) dodecahedron, (e) icosahedron) an electric charge $+q$, whereas into the center of the solids a balancing negative charge, q times the number of vertices. Let us rotate the objects with angular velocity ω around the axes indicated in the figure. The motion is non-relativistic.



    Which is the lowest non-vanishing multipole moment in each case, and how does the total power of the emitted electromagnetic waves depend on the frequency (calculated in leading order), if the frequency ω is small? Based on this, find an analogous argument stating that the 'infinite' regular solid, the sphere, rotated around one of its symmetry axes, will not emit any electromagnetic radiation.

(Máté Vass)

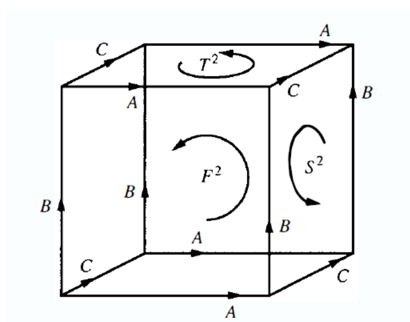
31. Two infinitely long straight wires of negligible radius are located at a distance d. The angle between their directions is α. The currents flowing in the wires are I_1 and I_2. Determine the force acting between them. Investigate the limit $\alpha \to 0$. How can you get the force between the two parallel wires, which corresponds to the old definition of the unit Amper?

(Gábor Széchenyi and Géza Tichy)

32. Let λ be the electrical resistance per unit length of a homogeneous, thin wire, with length L. This wire forms an ellipse. We can connect a known electrical potential difference U between any two points of the conductor frame. Develop a measurement procedure to determine the geometric parameters of the ellipse with only a resistance meter and a magnetometer being available. Give a specific numerical solution using parameters of your choice. Let's be creative.

(Gábor Homa and Ábel Tóth)

33. A renowned researcher at the Extra-Terrestrial Physical Society plans to submit an innovation project proposal, with the title *'Eternal electricity at eternal cost'*, in which the innovative applications of keeping a constant current for an infinite amount of time in a closed circuit is discussed. His friend has recently published his latest result, worth of a Nobel prize, demonstrating that their world is in reality a 3-torus, T^3. Being educated in topology, he warns our researcher, that due to the topology of their world, there can be a closed curve, along which – independently of the possibilities of realization – it is physically impossible to keep a constant current running for an infinite amount of time. Let us help our researcher to prove the statement above. That is, consider the Maxwell equations on a 3-torus, and find closed loops along which it is impossible to keep the current for an infinitely long time. Why does this problem not exist in our $\mathbb{R}^3$ world? Let us name, or construct, a three dimensional manifold, where problems similar to the 3-torus arise!



Hint: The 3-torus can be depicted as in the figure above. The opposite faces of the cube are 'glued' together, that is, if we exited on one face, we enter on the other. The faces of the cube (T^2, S^2, F^2) are all the usual, doughnut-shaped 2-tori. Be careful, that the 3-torus has no edge! It is useful to look up de Rham's theorem.

(Máté Vass and János Takátsy)

34. We take a side-on photograph of the front wheel of a relativistically moving bicyle. The wheel contains 20 radial spokes with negligible thickness. How does the photo look like if we use short exposure? Beyond the Lorentz transformation take into account the time delay of the light rays arriving to the observer from the spokes. What spots of the photos can be seen sharp? Investigate also the classical limit.

(Gábor Széchenyi)

35. A starship moves along a straight trajectory, and during the proper time interval $d\tau$, it ejects propellant gas of amount $dm = \mu(\tau)d\tau$ backwards. The relative velocity between the spacecraft and the propellant is constant by the construction of the engine, it is $u = c/n$ where $n > 1$. The captain of the ship can – within sensible limits – control the function $\mu(\tau)$ (before running out of fuel).

The passengers wish to travel conveniently, therefore they request a constant acceleration, corresponding the terrestrial $1g$, to be set by the captain. Reaching half way after proper time T, the captain turns the engines off for a brief period, flips the spaceship, and from then on the ship decelerates again with a constant rate $1g$ until reaching the destination.

What is the distance to the destination star from Earth? How long did the travel last, as seen from Earth? Which is the fraction of the mass of the spaceship remaining, relative to the initial, upon arrival?

Let us make as well an estimation using numerical values. Assume the total 'useful mass' of the spaceship, which arrives (including passengers, pets, robots, videos, helicopters, food, beer, seeds, etc) to be 1000 tons. Set the parameter n to be 5, or 2! Assume that the total travel time $2T$ takes 10 terrestial years. Give the numerical values for the questions above! What was the initial mass of the spaceship during launch?

Study the limiting case of 'photon propulsion'!

Hint: Use the unit system where $c = \hbar = 1$. Then the unit of time is approximately 1 year, the unit of length is about 1 light-year.

(Gyula Dávid)

36. According to some new theories of particle physics electrons can interact with the hypothetical 'dark photons', the lightest particles of the famous dark matter. The quantum numbers of dark photons are identical to the ordinary photons but they have non-vanishing rest mass. In the beginning of our story an electron absorbs a dark photon with velocity of opposite direction. After this the excited electron emits an ordinary photon, which decays to an electron-positron pair. The members of the pair and the original electron fly with velocities of same absolute value.

a) What is the maximal velocity of the dark photon causing this sequence of events?

b) What is the rest mass of the dark photon if it arrives with this maximal velocity?

Hint: Use the unit system $c = 1$.

(Gyula Dávid)

37. It is trivial, that unlike bulk viscosity, shear viscosity (i.e. the shear stress tensor) is zero in nonrelativistic hydrodynamics if the flow is one dimensional.

What about relativistic hydrodynamics? Are the components of the shear stress tensor zero, if

a) the flow is one dimensional in three dimensional space (i.e. two spatial components of the four-velocity are zero and the third spatial component only depends on the corresponding coordinate);

b) or if there is only one spatial dimension?

Compare the relativistic and nonrelativistic cases, and if there is a difference, try to trace its physical origin.

(Máté Csanád and Márton Nagy)

38. What is the physical meaning of the eigenvalues and eigenvectors of the energy-momentum tensor T_{kl} describing the energetic properties of the free electromagnetic field?

Hint: Use the answer for the similar question in the case of the electromagnetic field strength tensor $F_{kl} = \partial_k A_l - \partial_l A_k$ where the four-vector A_k is the electromagnetic four-potential (see Problem 17. in Ortvay Contest 2015). Treat carefully the degenerate cases.

(Gyula Dávid)

39. In a parallel universe, the scientists found out that Maxwell was incorrect, and that the equations of electrodynamics are nonlinear. The new Lagrangian proposed for the description of the free electromagnetic field is the following:

$$\mathcal{L} = -\frac{E_0^2}{2}\sinh\left(\frac{F_{kl}F^{kl}}{2E_0^2}\right),$$

where F_{kl} is the usual electromagnetic field strength tensor, and E_0 is a new physical constant, having dimension of square root of energy density.

How do the new 'material equations' of the new electrodynamics will look like? Study the properties of 'electromagnetic' plane waves in this theory! What new phenomena will emerge, if we exchange the function sinh to function sin?

(Gyula Dávid)

40. In a parallel universe, Einstein gave up the research career in time for being a programmer in a high profile corporation, therefore, he did not manage to construct the gravitational equation named after him. Fortunately, he did work out the theory of special relativity. The thus weakened physicist society came up with the following concept of the gravitational phenomena.

The Newtonian gravitational basic equation ($\Delta\Phi = 4\pi G\varrho$) has been changed to a covariant dynamical equation, such that a symmetric tensor $h_{\mu\nu}(x)$ was introduced, satisfying the following equation: $\Box h_{\mu\nu} = 8\pi G T_{\mu\nu}$, where $T_{\mu\nu}$ is the energy-momentum tensor. The action corresponding to the equation above takes the form (without indices) $S_1 \sim \int d^4x \left(\frac{1}{G}\partial h \partial h + hT\right)$. (Complete the indices, and also add the numerical factors!) Denote the usual Minkowski-like metric tensor by $\eta_{\mu\nu}$ with signature $(-,+,+,+)$, which can be used to 'pull up and down' the indices. Calculate the symmetric energy-momentum tensor corresponding to the action S_1 above. Let us prove, that this is not the same as T, and add the missing term. The new action can be denoted as S_2. Continue this procedure! Does it leaad to the complete Einstein-Hilbert action? Starting by the tensor $h_{\mu\nu}(x)$ of the gravitational field theory how can we get the metric tensor $g_{\mu\nu}(x)$ of the general relativity theory? What is the physical reasoning behind the need to complete the original energy-momentum tensor?

(Máté Vass)

41. Assume the two parameter cosmological Einstein–Fridman–de Sitter–Lemaitre–Robertson–Walker equation describing the expanding Universe to be a usual dynamical (evolutionary) equation, and study the time dependence of the states in the parameter space of the system. Let us restrict ourselves to the realm which is relevant to the present Universe, with equation of state $p = 0$ – in this case, the energy content of a given material element is conserved.

The main question of cosmology is the determination of the scaling function $a(t)$, describing how the galaxies drift away, and the EFdSLRW equation system is relevant for this. However, the function $a(t)$ can not be directly measured. Thus consider the coordinates of our state space be the following x and y, expressed by locally measurable quantities: $x = H(t)\sqrt{3c^4/8\pi G\varepsilon(t)}$ and $y = 3\Lambda/8\pi G\varepsilon(t)$. Here c is the speed of light, G is the gravitational constant, Λ is the cosmological constant concievend by Einstein, $H(t)$ is the momentary Hubble constant, and finally $\varepsilon(t)$ is the current full energy density of the Universe. Both x and y can be positive or negative (why?).

Place the separating lines of the various cosmological models with different character on the plane of the variables x and y, that is, determine the limiting curve between closed and open solutions, and between the accelerating, and decelerating expansion. Find the attractive and repulsive fix points of the development equation, the attractive and repulsive sets, attractors, repellors, and study the stability of various solutions of the evolution equation. Based on this, draw the trajectories of the cosmic developments with different storyline on the plane (x, y). (It is preferable to make a simple, hand-drawn sketch with sufficient explanations, rather than a computer imagery without explanations). Show qualitatively the functions $a(t)$ corresponding to the various domains of the figure. Where do the well known single parameter family of Friedmann's models appear in the above plane?

(Gyula Dávid)

42. Study the simplified model of the Solar System, consisting only of the Sun and the Jupiter. How much energy is radiated in form of gravitational waves during one full orbit of the planet? Estimate the time, after which the Jupiter loses its energy, and spirals into the Sun! Compare this timescale to the relevant astrophysical and particle physical characteristic time constants! Repeat this calculation and estimation for the full Milky Way!

(Dávid Gyula)

43. A quantum particle moves in one spatial dimension, and beyond the usual physical quantities it has an additional internal degree of freedom which can be described by a bivalued variable. The Hamiltonian of the system is the following (use the unit system $\hbar = 1$):

$$\hat{H}(\hat{x}, \hat{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \hat{x} & \hat{p} \\ \hat{p} & -\hat{x} \end{pmatrix},$$

where $\hat{x}$ and $\hat{p}$ are the usual position and momentum operators.

a) Calculate the eigenvalues and eigenvectors of the Hamiltonian.

b) What is the time evolution operator of the system? Express it by the usual creation and annihilation operators in the simplest form.

(Gyula Dávid)

44. After the unfortunate Fukushima Daiichi nuclear disaster, the radioactive contamination caused a mutation in the Ocean wildlife. A group of British scientists led by Sir Hamilton found a new mollusc species. The race shows many similarities with the order of octopoda, but instead of eight limbs this creature can have N identical legs. On every leg an equal number (M) of suction cup cells is located. The zealous scientists showed that between the cup cells and the brain (which contains one cell also) information can only flow by an electron hopping cell by cell via a thin quantum field. The new mollusc is named after this phenomenon: *Polypus quantus* (f.m.: *Polypus quanta*). Sadly, because of the fast evolution, the poor creature cannot find its proper values. Let's help him and find an analytic expression for all of its energy eigenvalues and eigenvectors.

Hint: the scientists examined a torn limb and made their measurements. They found that all the on-cell energies are the same on every cell and the hopping between the cells are also independent of location.

(Colin Lambert, Zoltán Tajkov and János Koltai)

45. Consider the following action functional with $r > 0$:

$$S[r] = \int dt \left(\frac{1}{2} m\dot{r}^2 - V(r) \right), \quad V(r) = \frac{\lambda}{r^2}.$$

a) Prove that the action above is scale invariant, that is, the transformation $r(t) \to s^\alpha r(st)$ is a symmetry of the system (α is to be determined, $s \in \mathbb{R}^+$). Determine the corresponding Noether-charge (denote it by $\mathcal{D}$).

b) Perform the canonical quantization of the system, and calculate the commutator $[\mathbf{H}, \mathcal{D}]$. Explain the result!

c) Write down the time-independent Schrödinger-equation corresponding to the system, in position representation, and provide the behaviour of the solution in the limit $r \to 0$! Study the other limiting case as well, when $r \to \infty$. What happens, if $2m\lambda < -1/4$? Give an explanation of the result.
Hint: find the probability current density in the first limiting case.

(Vass Máté)

46. Consider a two dimensional Sierpiński-triangle shaped solid state object. Figure (a) illustrates the iteration process of the Sierpiński triangle, whereas the fractal appears after an infinite number of iterations.

Let us place pointlike bodies with mass m into each vertex, and springs with constant k on each edge, and use periodic boundary conditions! Determine the temperature dependence of the heat capacitance of the resulting solid state model when $T \to 0$, and determine the equation of state of the phonons.

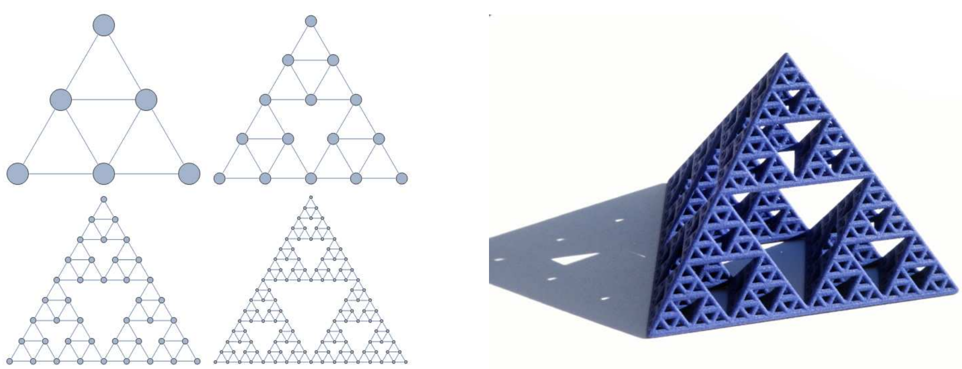
Hint: Since the problem can be related to a random walk on a fractal, it is sufficient to consider the scalar version of the problem, that is, the spring force can be taken to be one dimensional (everywhere). The movement of the i-th point may then be described with the equation $m\ddot{u}_i = k\sum_j(u_i - u_j)$, where the summation runs over all the neighbouring points. Besides, it is useful to remember the Debye model.

With that in hand, let us try to generalize the problem to the d Eucledian dimensional version of the Sierpiński-triangle (see panel (b) of the figure for the 3D version).

Let us assume, that we place a spin to all the vertices of the resulting fractal. Such a spin is described by a two component vector, with unit length, and can point to any direction on the unit circle (that is, it does not only have values of ± 1). Take the following usual interaction between the spins

$$H = -J\sum_{<ij>} \mathbf{S}_i \mathbf{S}_j,$$

where the spins can interact only with their immediate neighbours. Will there be a ferromagnetic phase in the 2D case? What about in higher dimensions (such as the Sierpiński-pyramid)?



a) b)

(Máté Vass)

47. Consider an assembly of N electrons, which are coupled with the same strength to the same (and only) electromagnetic mode. The mode is linearly polarised.
Using exclusively the dipole approximation, the Hamiltonian can be written as:

$$H = \sum_{k=1}^{N} \frac{1}{2m}\Big[\mathbf{p}_k - e\mathbf{A}\Big]^2 + \hbar\omega(N + \frac{1}{2}), \quad \text{where} \quad \mathbf{A} \approx \sqrt{\frac{\hbar}{2\omega_\mu V\varepsilon_0}}\mathbf{e}_\mu(a_\mu + a_\mu^+).$$

Find the energy eigenstates. What kind of, appropriate measure of the disorder of the N particles in momentum space can be defined? How does it affect the energy eigenvalues?

(Ákos Gombkötő)

48. A pair of scientists implement the following experiment: Far away from any concernable gravitation, they prepare a maximally entangled state

$$\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)^M$$

using two modes of bosonic, massless, scalar-field, such that one detector measures the A, the other detector measures B subsystem. The index M denotes that the zero/one-particle states are to be understood in Minkowski space.

The two detectors are comoving very closely to each other, and approach a black hole together. Then the detector A follows a geodetic, and enters the event horizon, while B accelerates relative to the geodesic, and do not enter. Estimate the measure of entanglement at the moment of the relative acceleration!

Hint: Use the Rindler-coordinates for the calculation, and take the infinite acceleration limit. The $|0\rangle_B$ Minkowski-vacuum state can be decomposed on the modes corresponding to the Rindler-space as:

$$|0\rangle_B \propto \frac{1}{\cosh\epsilon}\sum_{n=0}^{\infty}\tanh^n\epsilon\; |n\rangle_B^I|n\rangle_B^{II}$$

where I, II are denoting the regions of the Rindler-space. Furthermore

$$\cosh\epsilon = (1 - e^{-\frac{C}{a}})^{-1/2},$$

where $C > 0$ is a mode-dependent constant, a is the acceleration.

(Ákos Gombkötő)

49. As a generalization of the one-particle-irreducible (1PI) effective potential of a quantum field theory of an arbitrary Φ field-variable, the two-particle-irreducible (2PI) version is given by the following two-variable functional:

$$V_{\rm 2PI}[\bar{\Phi}, G] = V_{\rm cl}[\bar{\Phi}] + \frac{i}{2}\int_k \mathrm{Tr}\log G(k) - \frac{i}{2}\int_k \mathrm{Tr}[G_0^{-1}(k)G(k)] + iV_2,$$

whose vanishing derivatives with respect to its first and second variables are equivalent to the field equation of a homogeneous condensate, and the Dyson equation of the propagator (G), respectively. Here $V_{\rm cl}$ is the classical potential, G_0 is the tree-level propagator matrix in a $\bar{\Phi}$ background, V_2 in turn is the sum of all two-particle-irreducible Feynman diagrams, which are made of vertices of the shifted classical action $S[\Phi] \to S[\bar{\Phi} + \Phi]$ and lines of G.

Let us construct the 2PI effective potential of an $O(N)$ symmetric theory of a $\phi^T = (\phi_1, \phi_2, ...\phi_N)$ field variable at the next-to-leading order of the $1/N$ expansion, defined via the following Lagrangian:

$$L = \frac{1}{2}\partial_\mu\phi^T\partial^\mu\phi - \frac{1}{2}m^2\phi^T\phi - \frac{\lambda}{24N}(\phi^T\phi)^2,$$

i.e. draw all Feynman diagrams that contributes to V_2. (Do not deal with renormalization.)

Hint: let us use an equivalent representation of the theory, where the dynamical variables are extended with an auxiliary field α, given by the following Lagrangian:

$$L' = \frac{1}{2}\partial_\mu\phi^T\partial^\mu\phi - \frac{1}{2}m^2\phi^T\phi - \frac{1}{2}\alpha^2 + i\sqrt{\frac{\lambda}{12N}}\alpha\phi^T\phi.$$

For this representation, in the next-to-leading order of the $1/N$ expansion, V_2 only contains one single diagram. Eliminate from $V_{\rm 2PI}$ the auxiliary variable α and its propagators using the aforementioned stationary conditions, and based on the results, give all the diagrams in question!

(Gergely Fejős)

50. Dr. Absoluto Zero, the Eternal President (Father of Homeland, Friend of the People, Lord Protector of the Gummy Palms etc) of the small but proud country on the Equator, Gummy Coast, has summoned his main court physicist. Dr. Ali Knowsbut Whatfor walked worried across the main square of Port Gummy. He glanced briefly on the tower of the Fourlargest Fortress [*strange name, obviously – the Author*], which had no demonstrative item hanging for the time being. What kind of fantastic structure is to be constructed today – or, alternatively, what wild idea of the dictator is to be explained him to be impossible to realize?

The dictator welcomed him with a broad smile.

"You certainly know, that next year, the 40000th anniversary of the Establishment of the Gummy Coast Republic will be commemorated." [*Yes, forty thousand. You have read it correctly.–the Author*] "My glorious ancestor and great predecessor, Absoluto Minus Thousand Seven Hundred Twenty Nine" [*which, by the way, happens to be the first absolute Euler pseudo-prime*] "achieved this great deed, which we need to celebrate properly. Let us place the image of the State Founder above our country. Any time when a member of our laborious people raises his head (which happens quite often in these happy days), should see the bold smile of the glorious ancestor, and his caring glance. Certainly, by the side of the Founder, one must have the image of the contemporary person who made this country such flourishing – you know who I mean."

The Court Physicist knew whom he meant. He asked something different rather.

"But Mr. President, how do you wish to place the images? Should it be a high tower?"

"Definitely not. There is a lot of such stuffs existing, and you have constructed such tower as well." [*See Problem 5 in Ortvay Contest 1993*]. "This should be something much bigger, with the images kept steadily above the people! Certainly, outside of the atmosphere, so that no rain, snow, guano, or drones will not degrade the glory."

"Yes, Mr. President must be thinking of the geosynchronous orbit! We will place an order tomorrow for a position, and then soon send the images up there" – replied the Court Physicist happily, since he has considerable interest in the geo-synchronous orbit industry after the Heaven Seven project. [*H7, see Problem 6 in Ortvay Contest 1987.*)]

"Wait a second. How high is the synchronized swimmer orbit or whatever?"

"Well, this I know precisely: 35786 kilometers from sea level of Port Gummy Harbour."

"You did not get it, man. I said forty thousand. Would you like to reduce the glorious history of our labourous people by thousands of years? You know who tried to do so last time – the Court Historian – and you have seen what happened, haven't you?"

The Court Physicist did see it, on display on the tower of the Fourlargest Fortress, so had no more questions to the numerical value.

"Mr. President, if we wish to keep the images at 40000 km altitude, it will require strong rockets and a lot of fuel."

"Certainly the gummy palm oil distiller factory, named after Dr. Absoluto Zero, is at your disposal. From tomorrow, it will produce exclusively rocket fuel."

"Mr President, this is a beau geste, but the gummy oil produced by this factory, which has good smell and look, has a small problem: if it gets to the atmosphere, ruins the gummy palm trees. It most only be used far from the Earth's athmosphere."

"Sure, spray the gummy gases outwards, so that it will not get back to Earth ever. At least we can claim to the Purple Party, or whatever, Green Party, that we do not pollute Earth. However be sparing! Program the rockets so that the speed of the emitted gas is of minimal velocity! If you use more than needed from the precious industrial material of our dear homeland, I will make you pay!" [*About the economocal importance of gummy palms se Problem 1 in Ortvay Contest 2011.*]

The Court Physicist started to make the calculations, when he heard the most surprising and final condition from the President:

“This is just the beginning! Our glorious homeland is just 40000 years old, so the initial altitude needs to be 40000 kilometers. But as each single day brings us further, with the sky as limit, the images must be ascended by 1 kilometer each day, such that all conditions before are fulfilled.”

At this point, the President was leaning closer to the Court Physicist:

“I tell you why this project is so important for me. When I was a child, hunting for Gummy Squirrels in the Square-shaped Round Gummy Woods, once I met a very old magician. He predicted that I will have a glorious career. He said however, that this career will end on the day when I can not keep my image above my people, and raise it day by day. Well, strive to succeed!”

The Court Physicst did strive to reach the goal. He know that the very old magicians of the Square-shaped Round Gummy Woods always tell the Truth. Soon the images, visible from thousands of kilometers, were ready, and were lifted on the anniversary of the foundation of the Republic. Once in outer space, the huge rockets, keeping the images in place were switched on, spreading gummy palm oil fumes all over the Solar System. The gummy palms shed the oil, shuttle spaceships were continuously carrying supply fuel, and the rockets, pre-programmed properly, were doing their job. As to the wish of the President, the exhaust gas speed was optimized, and the orbit altitude was daily adjusted. Since in the realm of Gummy Coast, there is Order, so nothing runs out, nothing breaks, nothing gets stuck... and ever since, if the labourous people lift their heads, they see (day by day higher and higher) the caring eyes of the Founder and the Father of Homeland.

Finally, the question is this: after the ceremonial launch of the images, how many days will the glorious Absoluto Zero continue to rule?

(Gyula Dávid)

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